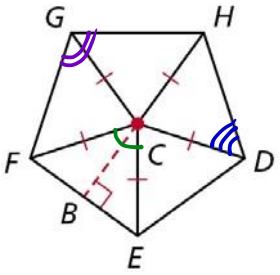
Notes: Area of Regular polygons

The **center** of a regular polygon is equidistant from the vertices.
The **apothem** is the distance from the center to a side. A **central** angle of a regular polygon has its vertex at the center, and its sides pass through consecutive vertices. Each central angle measure of a regular n-gon is $\frac{360^\circ}{n}$.

Regular pentagon *DEFGH* has a center *C*, apothem *BC*, and central angle $\angle DCE$.



Find:

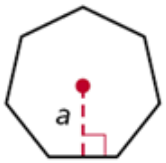
$m\angle FCE = \frac{360}{5} = 72^\circ$ (Central angle)

$m\angle FGH = \frac{(n-2)180}{n} = \frac{(5-2)180}{5} = 108^\circ$ (interior angle)

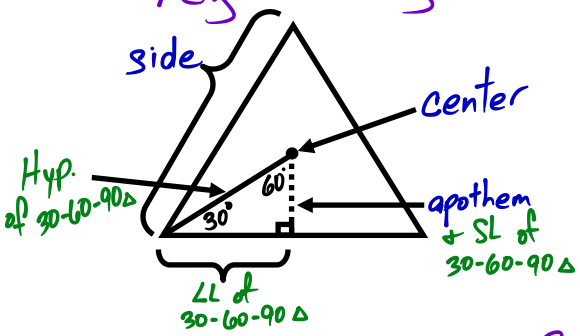
$m\angle HDC = \frac{108}{2} = 54$ (divide by 2)

Area **Regular Polygon**

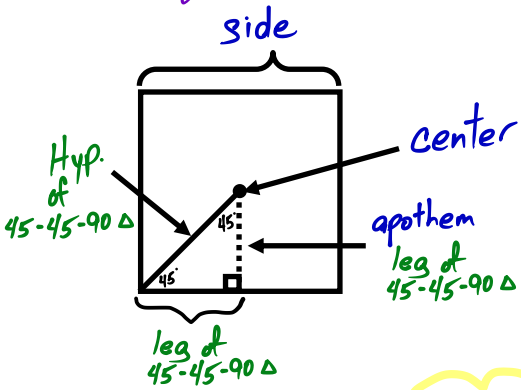
The area of a regular polygon with apothem *a* and perimeter *P* is $A = \frac{1}{2}aP$.



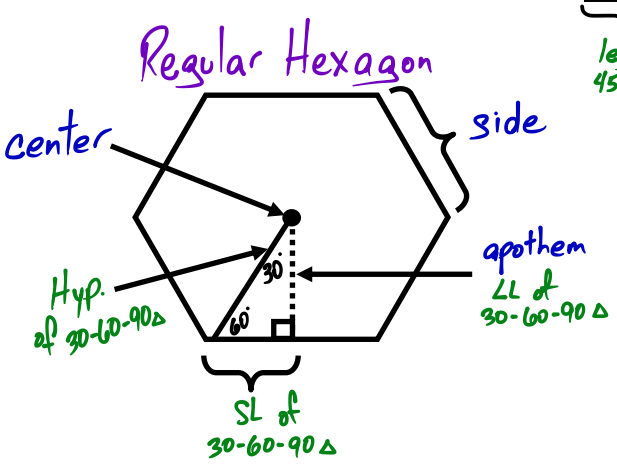
Regular Triangle



Square



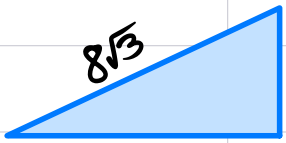
Regular Hexagon



SL	LL	Hyp
X	$X\sqrt{3}$	2X

leg	leg	Hyp
X	X	$X\sqrt{2}$

Review: 30-60-90 triangles



SL	LL	Hyp
x	$x\sqrt{3}$	$2x$
		$8\sqrt{3}$

times $\sqrt{3}$ ↑ divide by 2 ↓

$\frac{2x}{2} = \frac{8\sqrt{3}}{2}$

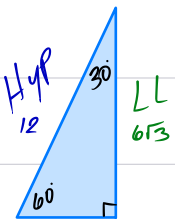
$x = 4\sqrt{3}$

$LL = x\sqrt{3}$

$= 4\sqrt{3}\sqrt{3}$

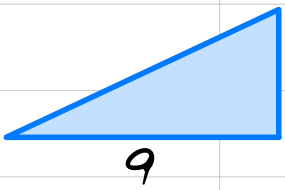
$= 4 \cdot 3$

$= 12$



SL	LL	Hyp
x	$x\sqrt{3}$	$2x$
6	$6\sqrt{3}$	12

times $\sqrt{3}$ ↑ times 2 ↓



SL	LL	Hyp
x	$x\sqrt{3}$	$2x$
	9	

divide $\sqrt{3}$ ↓ times 2 ↑

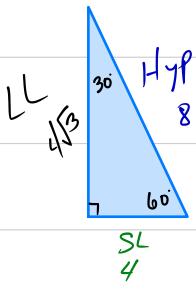
$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{9}{\sqrt{3}}$

$x = \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

$Hyp = 2x$

$= 2(3\sqrt{3})$

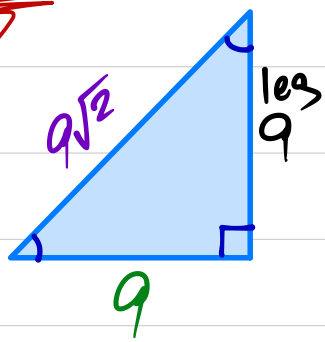
$= 6\sqrt{3}$



SL	LL	Hyp
x	$x\sqrt{3}$	$2x$
4	$4\sqrt{3}$	8

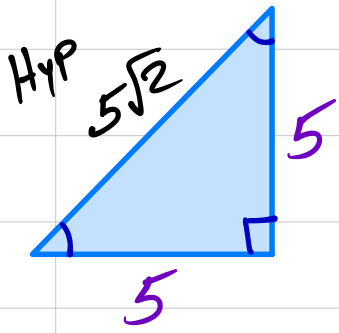
divide $\sqrt{3}$ ↓ times 2 ↑

Review: 45-45-90 Δ



leg	leg	Hyp
x	x	$x\sqrt{2}$
9	9	$9\sqrt{2}$

same → ↑ times $\sqrt{2}$ ↑



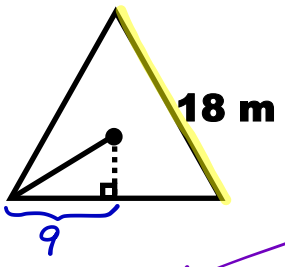
leg	leg	Hyp
x	x	$x\sqrt{2}$
5	5	$5\sqrt{2}$

divide by $\sqrt{2}$ ↓

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}}$

$x = 5$

EXAMPLE 1 Find the area of the equilateral triangle.



x	$x\sqrt{3}$	2x
$3\sqrt{3}$	9	$6\sqrt{3}$

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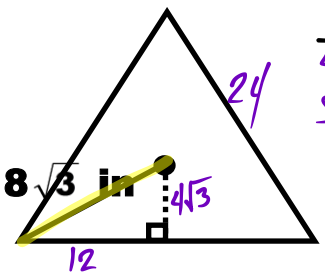
$a =$ ___
$s = 18$
$P = 18 \cdot 3 = 54$

$A = \frac{1}{2}aP$
$= \frac{1}{2}(3\sqrt{3})(54)$
$= 81\sqrt{3}\ m^2$
$\approx 140.3\ m^2$



x	$x\sqrt{3}$	2x
$4\sqrt{3}$	12	$8\sqrt{3}$

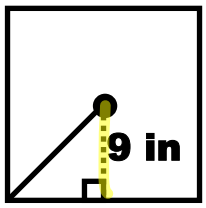
See previous page

$a = 4\sqrt{3}$
$s = 24$
$P = 24 \cdot 3 = 72$

$A = \frac{1}{2}aP$
$= \frac{1}{2}(4\sqrt{3})(72)$
$= 144\sqrt{3}$

Find the area of the given shapes:

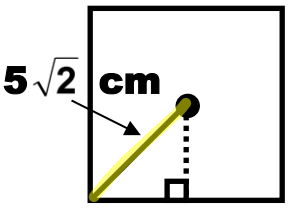
Ex.



leg	leg	Hyp
x	x	$x\sqrt{2}$
9		

Don't need hypotenuse
Square side $= 9(2) = 18$

$A_{\square} = s^2$
$= 18^2 = 324\ in^2$



leg	leg	Hyp
x	x	$x\sqrt{2}$
5	5	$5\sqrt{2}$

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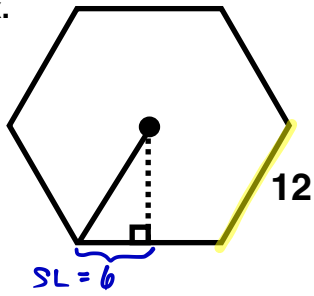
$A = \frac{1}{2}aP$

can also use $A_{\square} = s^2$
$= 10^2 =$ 100 cm²

Ex.

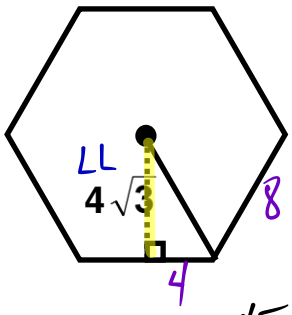


x	$x\sqrt{3}$	2x
6	$6\sqrt{3}$	12

See previous page

$a = 6\sqrt{3}$
$s = 12$
$P = 12(6\text{ sides}) = 72$

$A = \frac{1}{2}aP$
$= \frac{1}{2}(6\sqrt{3})(72)$
$= 216\sqrt{3}\ cm^2$
$\approx 374.1\ cm^2$



x	$x\sqrt{3}$	2x
4	$4\sqrt{3}$	8

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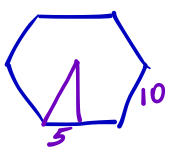
$a = 4\sqrt{3}$
$s = 8$
$P = 8(6\text{ sides}) = 48$

$A = \frac{1}{2}aP$
$= \frac{1}{2}(4\sqrt{3})(48)$
$= 96\sqrt{3}$
≈ 166.3

A regular hexagon has a perimeter of 60 cm. Find it s area.



$a = 5\sqrt{3}$
$s = 60/6 = 10$
$P = 60$
$SL = 10/2 = 5$

x	$x\sqrt{3}$	2x
5	$5\sqrt{3}$	

times $\sqrt{3}$

$A = \frac{1}{2}aP$
$= \frac{1}{2}(5\sqrt{3})(60)$
$= 150\sqrt{3}\ cm^2$
$= 259.8\ cm^2$